

Rose Petals for Tropical Cyclones

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Abstract

Determination of tropical cyclone tracks is of utmost importance for providing advance warning for cyclone preparedness. In this paper the equation of motion of a tropical cyclone under the action of two perpendicular landmasses with plane boundaries and an azimuthal wind has been derived. This has been found to be a rose petal with four loops. This shows reasonable agreement with the observed tracks of cyclones in the Bay of Bengal. It is possible to forecast both the landfall point and time with improved accuracy even after only a day's observation of the cyclones have been obtained.

Introduction

DETERMINATION of the tracks of tropical cyclones is one of those problems in meteorological science which has so far eschewed analytical treatment. It is possible to locate the position of the cyclone centre and estimate the intensity from satellite pictures and conventional data with certain accuracy¹⁻³. However, no matter how accurate and how extensive our observations are, they give only the present state of affairs of a cyclone. They do not directly say when and where a cyclone is going to hit the land. When a cyclone is formed say, in the Bay of Bengal, there is the possibility that it can hit any of the adjoining countries namely Sri Lanka, India, Bangladesh and Burma. During the initial stage of formation, all the countries have to remain alert. With such a state of affairs, it is very difficult to take adequate cyclone preparedness measures such as evacuation of people from the low lying coastal areas to safer places well ahead in time. Even in the advanced countries, where prediction is done using statistical and dynamical methods, seventy-two-hour advance prediction can go wrong by a maximum of one thousand nautical miles³. This shows the necessity of making improved models for the path prediction of tropical cyclones. In this paper, an attempt will be made to formulate a model of cyclone motion using hydrodynamics and classical mechanics.

Theory of the Cyclone motion in the Bay of Bengal

The Bay of Bengal is bounded by almost two plane perpendicular coastal boundaries intersecting at the Meghna Estuary of Bangladesh (Fig. 1). We assume the cyclone to be a cylindrical vortex and study the motion in two dimensions using the method of images in hydrodynamics. In the case of two dimensional nonviscous irrotational motion, the motion is represented by a complex potential given by $w = \varphi + i\psi$ where φ and ψ are velocity potential and the stream function respectively. Both φ and ψ satisfy Laplace's equation $\Delta^2\varphi = \Delta^2\psi = 0$. In polar co-ordinates, the velocity components are

$$q_r = -\frac{d\varphi}{dr} = -\frac{1}{r} \frac{d\psi}{d\theta}$$

$$q_\theta = -\frac{1}{r} \frac{d\varphi}{d\theta} = \frac{d\psi}{dr}$$

The cyclone vortex is supposed to have a radius 'a'. The vorticity inside the vortex has a constant value ω and outside, the vorticity is assumed zero. The velocity distributions inside and outside the vortex are given by (shown in Fig. 2)

$$v = \frac{1}{2}\omega r \quad (r < a)$$

$$v = \frac{1}{2} \frac{\omega a^2}{r} \quad (r > a)$$

This model is quite realistic and roughly corresponds to a real cyclone.

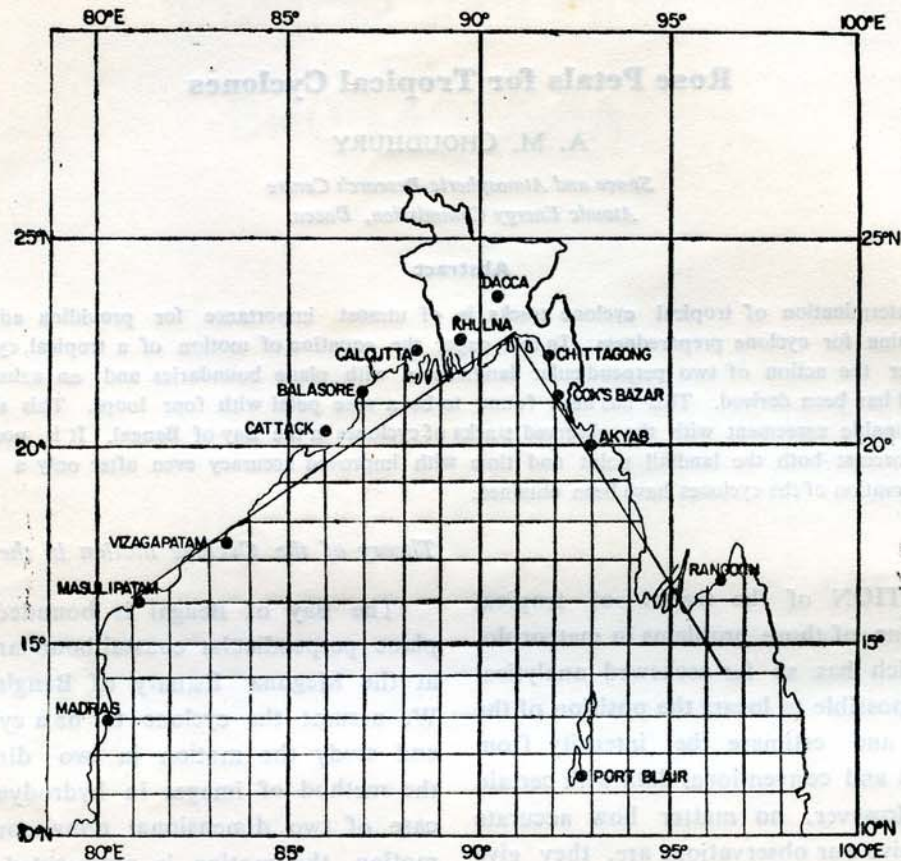


Fig. 1. The Bay of Bengal is bounded by almost two plane perpendicular boundaries intersecting at the Meghna Estuary of Bangladesh.

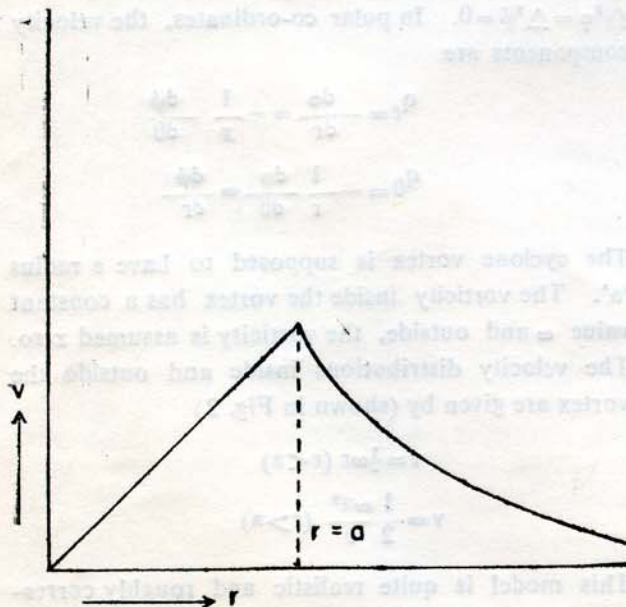


Fig. 2. Velocity Distribution in the vortex.

First we study the effect of the two orthogonal landmasses with plane boundaries and then introduce an azimuthal wind. The complex potential for a vortex of strength μ is $w = i\mu \log(z - z_0)$ where z_0 is the position of the vortex. The strength μ is given by $\mu = \frac{1}{2} \omega a^2$. The presence of the orthogonal plane boundaries will modify the motion and will be equivalent to a system of three images as shown in Fig. 3. The movement of the real vortex will depend on the effect of these image vortices and the azimuthal wind. The problem of the motion of a vortex filament parallel to two perpendicular planes is studied in books of hydrodynamics⁴. But here we are taking the radius 'a' to be finite to correspond to real tropical cyclones and in addition, an azimuthal wind is introduced to take account of other factors besides the boundary walls.

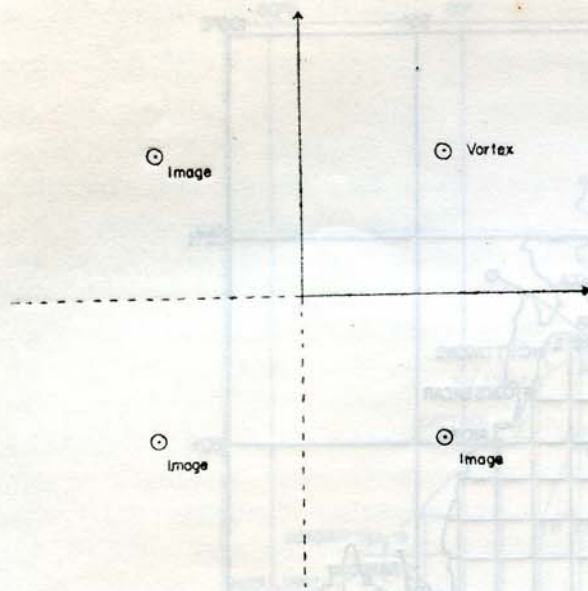


Fig. 3. System of Images of a Vortex due to the presence of two orthogonal plane boundaries.

The complex potential for the real and the three virtual image system is $w = i\mu \log(z^2 - z_0^2) - i\mu \log(z^2 - \bar{z}_0^2)$ where \bar{z}_0 is the complex conjugate of z_0 . Excluding the real vortex, whose effect on itself is nil, the complex potential is

$$w = i\mu \log(z^2 - z_0^2) - i\mu \log(z^2 - \bar{z}_0^2) - i\mu \log(z - z_0) = i\mu \log(z + z_0) - i\mu \log(z^2 - \bar{z}_0^2).$$

It can be shown that the velocity components in polar coordinates experienced by the real vortex are given by

$$q_r = \frac{\mu \cos 2\theta}{r \sin 2\theta} \dots (1)$$

$$q_\theta = -\frac{\mu}{2r} \dots (2)$$

We now introduce an azimuthal wind μ/r so that

$$q_\theta = -\frac{\mu}{2r} + \frac{\mu}{r} = \frac{\mu}{2r} \dots (3)$$

Dividing equation (1) by (3) and integrating we get $r = K \sin 2\theta$ where K is the integration constant. This equation is known as the equation of a rose petal with four loops (Fig. 4). This is the equation of

the trajectory of a tropical cyclone when the boundaries are two orthogonal planes and the landfall is the origin. The tangent to the curve at the origin serves as the x-axis.

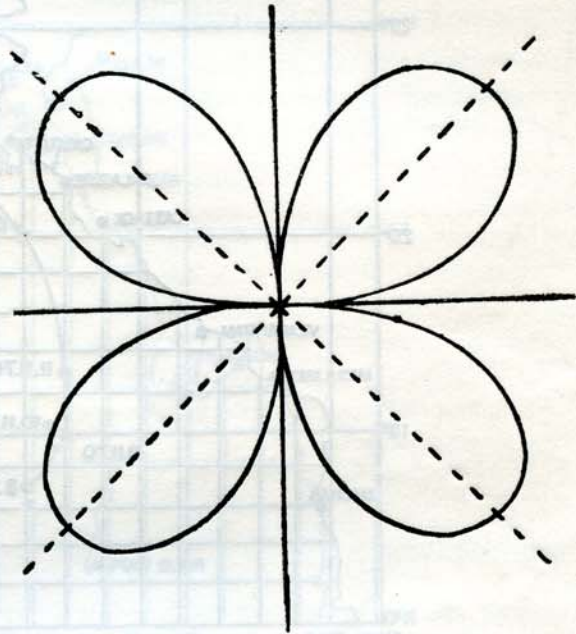


Fig. 4. A rose petal with four loops.

Observational Evidence :

Tracks of cyclones of the Bay of Bengal have been compiled from weather satellite APT pictures taken at Dacca for the period 1968-78 and ground observations wherever available. It has been found that these tracks agree quite well with the computed trajectories from the curve $r = K \sin 2\theta$ with landfall point as the origin. The fitting has been done for $0 \leq \theta \leq 45^\circ$ and this holds for both kinds of trajectories whether they curve eastwards or westwards. Details of observational evidences have been given in ⁵. The discrepancy between the observed and the computed points in general do not exceed 20%. A typical example is given in Fig. 5 showing the case of November 1970 cyclone. The circles represent the observed points while the crosses represent the computed points.

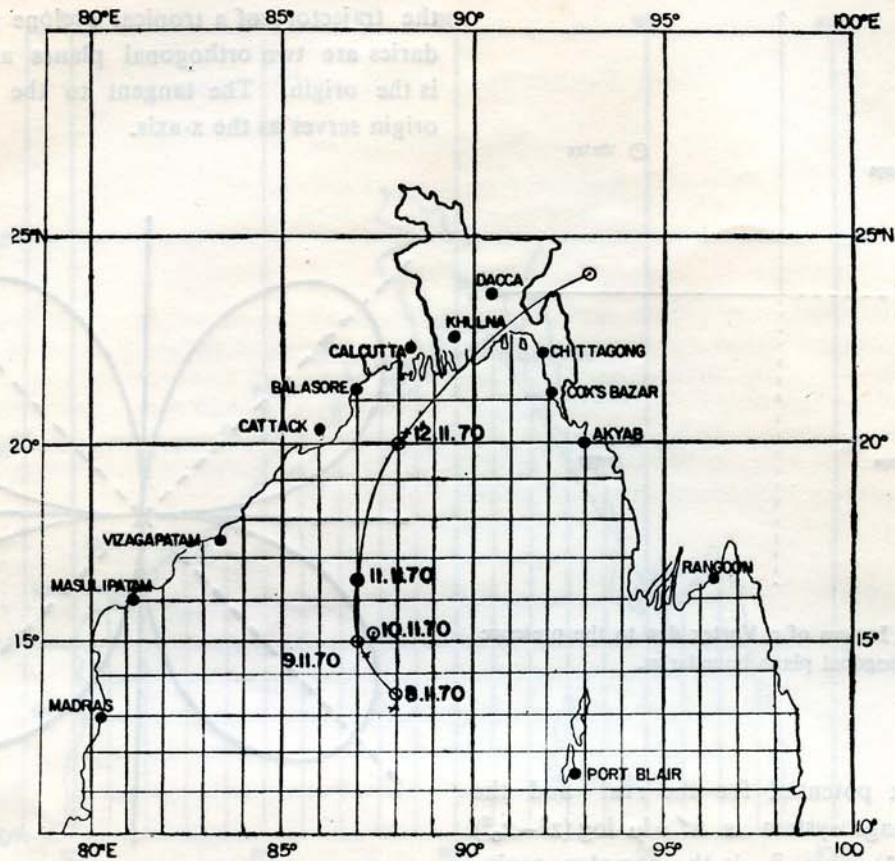


Fig. 5. Track of November 1970 cyclone : The circles represent the observed points and the crosses represent the fitted points for the rose petal.

Cyclone Track Forecasting :

As soon as we get more than two observation points we know the curvature of the track at the mean point. We also know the translational velocity of the cyclone from these observations. Now the radius of curvature ρ and the velocity v are given by

$$\rho = \frac{K(1 + 3 \cos^2 2\theta)^{3/2}}{\sin^2 2\theta + 8}$$

$$v = \frac{\mu(1 + 3 \cos^2 2\theta)^{1/2}}{2K \sin^2 2\theta}$$

μ can be estimated from satellite pictures. Hence these two equations determine θ uniquely.

We also know that $\tan \phi = \frac{1}{2} \tan 2\theta$ where ϕ is the angle the radius vector makes with the tangent. This determines the landfall point which is the

origin. The value of K can also be determined from the above two equations and so the curve $r = K \sin 2\theta$ can be completely traced out. As soon as we have another point, we can compute another curve which may differ slightly from the earlier curve and so on. Thus for each two consecutive observation points, we have a prediction for a landfall point. It has been found that the actual point is very close to the predicted landfall points. Fig. 6 shows the actual landfall points and the predicted landfall points for November 1970 cyclone. Thus it is possible to identify the region where a cyclone is going to hit some 2/3 days in advance in most cases. In a day we get three to four observation points and hence prediction can be made after a day's observation.

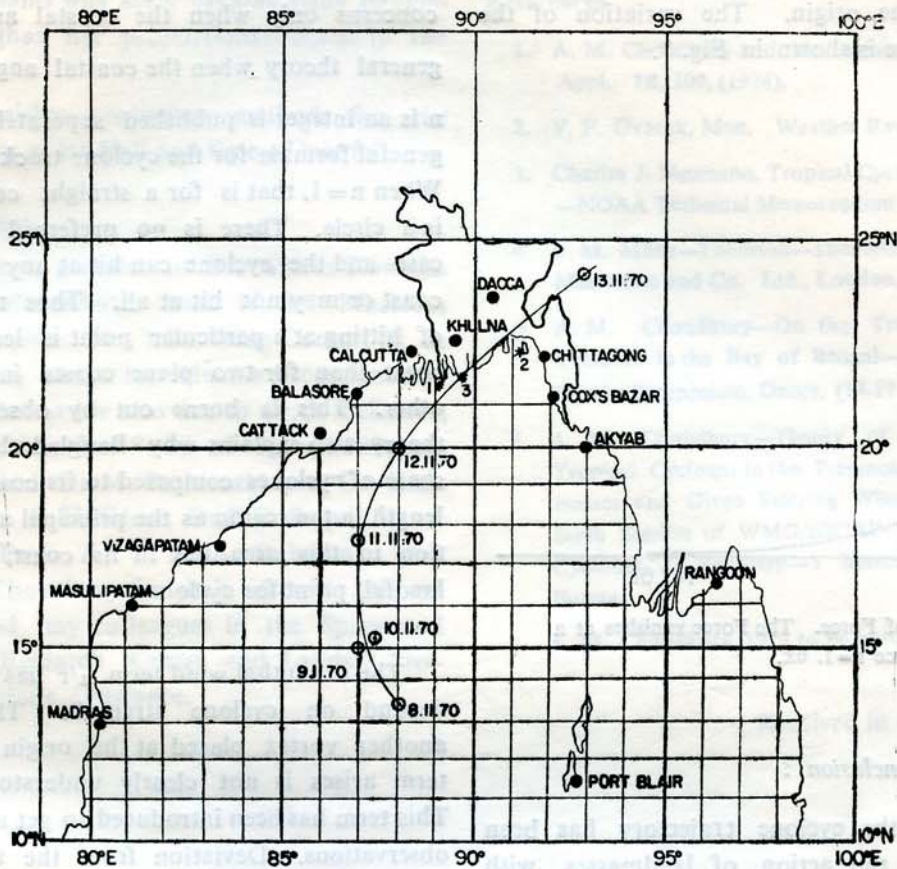


Fig. 6. Predicted landfall points for the November 1970 cyclone. The points 1*, 2*, 3* represent the predicted landfall points according to prediction made on 9.11.70, 10.11.70 and 11.11.70 respectively. It may be seen that the predicted points are quite close to the observed landfall.

The time of travel for the cyclone is given by

$$T = \frac{K^2}{\mu} \left(\theta - \frac{\sin 4\theta}{4} \right)$$

Thus it is possible to find out the time of landfall as well. Typical discrepancies between the calculated and the observed times of landfall for the November 1970 cyclone are +9 hrs, +23 hrs and 0 hrs as determined by this method after two, three and four observations where available (one observation per day) respectively.

Law of Force :

Once we know the trajectory, we can find out the law of force for the vortex motion. For the

trajectory $r = K \sin 2\theta$, this comes out to be

$$F = \frac{8K^2h^2}{r^5} - \frac{3h^2}{r^3}$$

where h is the angular momentum assumed constant. This is directed towards the origin. The force is infinite when $r=0$ i. e. at the landfall point. At $r=1.6K$ the force becomes zero. A cyclone is formed at a distance smaller than this value as the maximum distance of the cyclone from the landfall point is K . This means that as soon as a cyclone is formed, the attractive force becomes operative. Thus this law explains why a cyclone is accelerated as it approaches the landfall. The magnitude of the force drops sharply with increasing

distance from the origin. The variation of the force with distance is shown in Fig. 7.

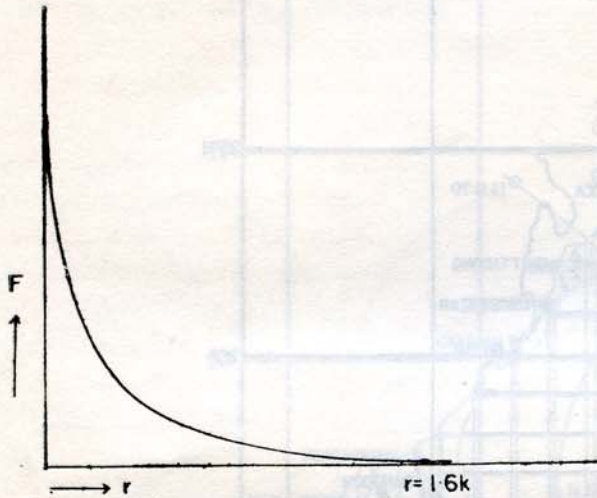


Fig. 7. Law of Force. The Force vanishes at a distance $r = 1.6k$.

Discussion and Conclusion :

Equation of the cyclone trajectory has been obtained under the action of landmasses with boundaries inclined perpendicular to each other and an azimuthal wind. The landmasses have been assumed to have plane boundaries in the hydrodynamical sense so that there is no flow of the liquid perpendicular to the boundary. This assumption may be valid when the cyclone is at a distance from the coast but breaks down at the landfall point. The boundary at the coast is only few feet high whereas the cyclone extends a few miles upwards. Of course, north of the coast, we have two branches of the towering Himalayas almost at right angles again. Thus the irregularities of the landmasses may also play some part in the cyclone motion. These factors may explain why a cyclone does not always hit the Bangladesh coast which is the principal angular formation in the region. Besides Bangladesh, The Bay of Bengal coast has angular formation at Sri Lanka-Southern India meeting point, near Madras, at Orissa-West Bengal coast, near Akyab and Rangoon in Burma. It seems that these coastal angles also play some part in cyclone movement. The present theory

concerns only when the coastal angle is 90° . A general theory when the coastal angle is $\frac{\pi}{n}$ where n is an integer is published separately⁶⁻⁷ and the general formula for the cyclone track is $r = K \sin n\theta$. When $n=1$, that is for a straight coast, the track is a circle. There is no preferred origin in this case and the cyclone can hit at any point on the coast or may not hit at all. Thus the probability of hitting at a particular point is less for a plane coast than for two plane coasts inclined at each other. This is borne out by observations. The theory also explains why Bangladesh has a greater share of cyclones compared to its coastal boundary length in the region as the principal angular formation in this area lies in its coast, which is the landfall point for cyclones.

The azimuthal wind term μ/r has been taken to depend on cyclone strength. This represents another vortex placed at the origin but how this term arises is not clearly understood physically. This term has been introduced to get agreement with observations. Deviation from the theory may be due to those effects not covered in this term. The theory may serve as a starting point to understand the cyclone movement from physical principles. For very strong cyclones, the fitting is very good whereas largest discrepancy arises for weak cyclones. It has been found that for some weak cyclones, two separate rose petals may be fitted, one for the initial motion and the other for the final motion, the cyclone making an abrupt change of course. This is probably due to some internal changes in the cyclone resulting in its change of strength and orientation. The observed and the predicted landfall points do not differ much in most of the cases. Time prediction for landfall is also correct within hours. In spite of theoretical inadequacies, the theory can explain the following :

- (1) It explains the observed tracks of cyclones in the Bay of Bengal.
- (2) It explains why a cyclone is accelerated when it approaches land.

- (3) It explains why more cyclone visits Bangladesh than her proportional share in the region.
- (4) It furnishes prediction methods for the forecast of landfall and time of landfall.

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